

Machten

Voor alle $a, b > 0$ geldt

$$1 \quad a^{-p} = \frac{1}{a^p}$$

$$2 \quad a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

$$3 \quad a^p \cdot a^q = a^{p+q}$$

$$4 \quad \frac{a^p}{a^q} = a^{p-q}$$

$$5 \quad (a \cdot b)^p = a^p \cdot b^p$$

$$6 \quad (a^p)^q = a^{pq}$$

Logaritmen

Voor alle $g > 0$ en $g \neq 1$ en alle $a, b > 0$ geldt

$$1 \quad {}^g \log ab = {}^g \log a + {}^g \log b$$

$$2 \quad {}^g \log \frac{a}{b} = {}^g \log a - {}^g \log b$$

$$3 \quad {}^g \log a^q = q \cdot {}^g \log a \quad (q \in \mathbb{I})$$

$$4 \quad {}^g \log a = \frac{{}^p \log a}{{}^p \log g} \quad (p > 0 \text{ en } p \neq 1)$$

abc-formule

Het oplossen van $ax^2 + bx + c = 0$, waarbij $a, b, c \in \mathbb{I}$ en $a \neq 0$.

Discriminant $D = b^2 - 4ac$.

$$\text{Als } D \geq 0 \text{ dan } x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

Als $D < 0$ dan geen reële oplossingen.

Goniometrische formules

$$1 \quad (\sin x)^2 + (\cos x)^2 = 1$$

$$2 \quad \tan x = \frac{\sin x}{\cos x}$$

$$3 \quad \sin x = \cos\left(\frac{1}{2}\pi - x\right)$$

$$4 \quad \cos x = \sin\left(\frac{1}{2}\pi - x\right)$$

$$5 \quad \sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y$$

$$6 \quad \cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$$

$$7 \quad \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$

$$8 \quad \sin(2x) = 2 \sin x \cdot \cos x$$

$$9 \quad \cos(2x) = (\cos x)^2 - (\sin x)^2 = 2(\cos x)^2 - 1 = 1 - 2(\sin x)^2$$

$$10 \quad (\cos x)^2 = \frac{1}{2}(1 + \cos 2x)$$

$$11 \quad (\sin x)^2 = \frac{1}{2}(1 - \cos 2x)$$

$$12 \quad \sin x + \sin y = 2 \sin\left(\frac{1}{2}(x+y)\right) \cdot \cos\left(\frac{1}{2}(x-y)\right)$$

$$13 \quad \sin x - \sin y = 2 \sin\left(\frac{1}{2}(x-y)\right) \cdot \cos\left(\frac{1}{2}(x+y)\right)$$

$$14 \quad \cos x + \cos y = 2 \cos\left(\frac{1}{2}(x+y)\right) \cdot \cos\left(\frac{1}{2}(x-y)\right)$$

$$15 \quad \cos x - \cos y = -2 \sin\left(\frac{1}{2}(x+y)\right) \cdot \sin\left(\frac{1}{2}(x-y)\right)$$

$$16 \quad \sin x \cdot \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

$$17 \quad \sin x \cdot \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$

$$18 \quad \cos x \cdot \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

Goniometrische vergelijkingen

$$\sin x = \sin \alpha \Leftrightarrow x = \alpha + k \cdot 2\pi \vee x = \pi - \alpha + k \cdot 2\pi \quad (k \in \mathbb{Z})$$

$$\cos x = \cos \alpha \Leftrightarrow x = \alpha + k \cdot 2\pi \vee x = -\alpha + k \cdot 2\pi \quad (k \in \mathbb{Z})$$

$$\tan x = \tan \alpha \Leftrightarrow x = \alpha + k \cdot \pi \quad (k \in \mathbb{Z})$$

Cyclometrische functies

$$y = \arcsin x \Leftrightarrow \sin y = x \text{ en } y \in \left[-\frac{1}{2}\pi, \frac{1}{2}\pi\right]$$

$$y = \arccos x \Leftrightarrow \cos y = x \text{ en } y \in [0, \pi]$$

$$y = \arctan x \Leftrightarrow \tan y = x \text{ en } y \in \left\langle -\frac{1}{2}\pi, \frac{1}{2}\pi \right\rangle$$

Graden en radialen

$$\alpha \text{ rad} @ \left(\alpha \cdot \frac{180}{\pi}\right)^\circ \quad \alpha^\circ @ \alpha \cdot \frac{\pi}{180} \text{ rad}$$

Differentiaalrekening

Rekenregels

$$1 \quad (c \cdot f(x))' = c \cdot f'(x), \text{ met } c \in \mathbb{R}$$

$$2 \quad (\alpha f(x) + \beta g(x))' = \alpha f'(x) + \beta g'(x), \text{ met } \alpha, \beta \in \mathbb{R} \quad (\text{somregel})$$

$$3 \quad (f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad (\text{productregel})$$

$$4 \quad \left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2} \quad (\text{quotiëntregel})$$

$$5 \quad f(g(x))' = f'(g(x)) \cdot g'(x) \quad (\text{kettingregel})$$

Standaardafgeleiden

$$1 \quad f(x) = c \Rightarrow f'(x) = 0$$

$$2 \quad f(x) = x^\alpha \Rightarrow f'(x) = \alpha \cdot x^{\alpha-1}, \text{ waarbij } \alpha \in \mathbb{R}$$

$$3 \quad f(x) = \sin x \Rightarrow f'(x) = \cos x$$

$$4 \quad f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

$$5 \quad f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$$

- 6 $f(x) = e^x \Rightarrow f'(x) = e^x$
- 7 $f(x) = a^x \Rightarrow f'(x) = a^x \ln a$
- 8 $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
- 9 $f(x) = {}^a \log x \Rightarrow f'(x) = \frac{1}{x \ln a}$
- 10 $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
- 11 $f(x) = \arccos x \Rightarrow f'(x) = \frac{-1}{\sqrt{1-x^2}}$
- 12 $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$

Integraalrekening

Rekenregels

- 1 $\int_a^b (\alpha f(x) + \beta g(x)) dx = \alpha \int_a^b f(x) dx + \beta \int_a^b g(x) dx$, met $\alpha, \beta \in \mathbb{R}$
- 2 $\int_{x=a}^b f(x) dg(x) = [f(x)g(x)]_{x=a}^b - \int_{x=a}^b g(x) df(x)$ (partiele integratie)

Standaardintegralen

- 1 $\int x^\alpha dx = \frac{1}{\alpha+1} x^{\alpha+1} + C$ ($\alpha \neq -1$)
- 2 $\int \frac{1}{x} dx = \ln|x| + C$
- 3 $\int \sin x dx = -\cos x + C$
- 4 $\int \cos x dx = \sin x + C$
- 5 $\int \tan x dx = -\ln|\cos x| + C$
- 6 $\int \frac{1}{\cos^2 x} dx = \tan x + C$
- 7 $\int a^x dx = \frac{a^x}{\ln a} + C$ ($a > 0$ en $a \neq 1$)
- 8 $\int e^x dx = e^x + C$
- 9 $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$
- 10 $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$

Numerieke wiskunde

Numerieke nulpuntsbepaling

Benaderen van een oplossing van $f(x) = 0$.

Het Newton-Raphson-proces

Startwaarde x_0 en $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, waarbij $n = 0, 1, 2, 3, \dots$.

Integratie

Benaderen van $I = \int_a^b f(x) dx$

Notatie: $h = \frac{b-a}{n}$, $x_0 = a$ en $x_k = x_0 + k \cdot h$ voor $k = 1, 2, \dots, n$.

Samengestelde trapeziumregel.

$I = T_n + E_T$, met

$$T_n = \frac{1}{2}h \{f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\}$$

$$\text{en } |E_T| \leq \frac{(b-a)^3}{12n^2} \max_{a \leq x \leq b} |f''(x)|.$$

Samengestelde regel van Simpson:

$I = S_n + E_S$, met

$$S_n = \frac{1}{3}h \{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\}$$

$$\text{en } |E_S| \leq \frac{(b-a)^5}{180n^4} \max_{a \leq x \leq b} |f^{(4)}(x)|.$$